1-2 Properties of Real Numbers

Main Ideas

- Classify real numbers.
- Use the properties of real numbers to evaluate expressions.

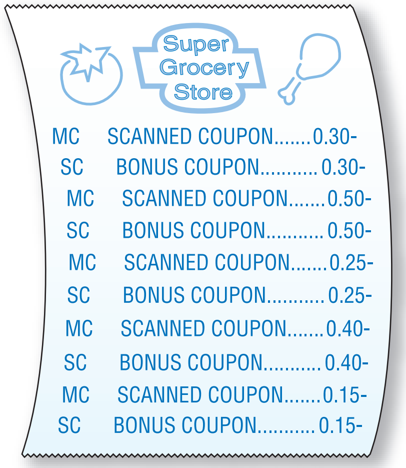
New Vocabulary

real numbers
rational numbers
irrational numbers

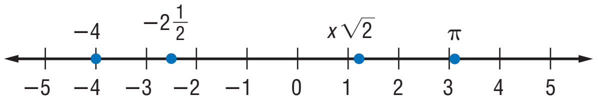
GET READY for the Lesson

Manufacturers often offer coupons to get consumers to try their products. Some grocery stores try to attract customers by doubling the value of manufacturers' coupons.

You can use the Distributive Property to calculate these savings.



Real Numbers The numbers that you use in everyday life are **real numbers**. Each real number corresponds to exactly one point on the number line, and every point on the number line represents exactly one real number.



Real numbers can be classified as either **rational** or **irrational**.

Review Vocabulary

Ratio the comparison of two numbers by division

KEY CONCEPT — *Real Numbers*

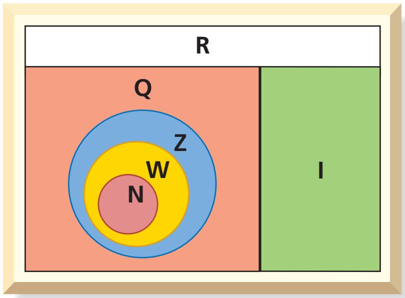
Words A rational number can be expressed as a ratio $\frac{m}{n}$, where m and n are integers and n is not zero. The decimal form of a rational number is either a terminating or repeating decimal.

Examples $\frac{1}{6}$, 1.9, 2.575757..., -3, $\sqrt{4}$, 0

Words A real number that is not rational is irrational. The decimal form of an irrational number neither terminates nor repeats.

Examples $\sqrt{5}$, π, 0.010010001...

The sets of natural numbers, {1, 2, 3, 4, 5, ...}, whole numbers, {0, 1, 2, 3, 4, ...}, and integers, {..., −3, −2, −1, 0, 1, 2, ...} are all subsets of the rational numbers. The whole numbers are a subset of the rational numbers because every whole number n is equal to $\frac{n}{1}$.



The Venn diagram shows the relationships among these sets of numbers.

R = reals   Q = rationals

I = irrationals   Z = integers

W = wholes   N = naturals

The square root of any whole number is either a whole number or it is irrational. For example, $\sqrt{36}$ is a whole number, but $\sqrt{35}$ is irrational and lies between 5 and 6.

Study Tip

Common Misconception

Do not assume that a number is irrational because it is expressed using the square root symbol. Find its value first.

EXAMPLE Classify Numbers

1 **Name the sets of numbers to which each number belongs.**

a. $\sqrt{16}$

$\sqrt{16} = 4$ naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

b. -18 integers (Z), rationals (Q), and reals (R)

c. $\sqrt{20}$ irrationals (I) and reals (R)

$\sqrt{20}$ lies between 4 and 5 so it is not a whole number.

d. $-\frac{7}{8}$ rationals (Q) and reals (R)

e. $0.\overline{45}$ rationals (Q) and reals (R)

The bar over the 45 indicates that those digits repeat forever.

CHECK Your Progress

1A. -185 **1B.** $-\sqrt{49}$ **1C.** $\sqrt{95}$

Properties of Real Numbers Some of the properties of real numbers are summarized below.

Reading Math

Opposites

$-a$ is read *the opposite of a*.

KEY CONCEPT *Real Number Properties*

For any real numbers *a*, *b*, and *c*:

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	If $a \neq 0$, then $a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$.
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	

EXAMPLE Identify Properties of Real Numbers

2 **Name the property illustrated by $(5 + 7) + 8 = 8 + (5 + 7)$.**

Commutative Property of Addition

The Commutative Property says that the order in which you add does not change the sum.

CHECK Your Progress

2. Name the property illustrated by $2(x + 3) = 2x + 6$.

EXAMPLE Additive and Multiplicative Inverses

3 **Identify the additive inverse and multiplicative inverse for $-1\frac{3}{4}$.**

Since $-1\frac{3}{4} + \left(1\frac{3}{4}\right) = 0$, the additive inverse of $-1\frac{3}{4}$ is $1\frac{3}{4}$.

Since $-1\frac{3}{4} = -\frac{7}{4}$ and $\left(-\frac{7}{4}\right)\left(-\frac{4}{7}\right) = 1$, the multiplicative inverse of $-1\frac{3}{4}$ is $-\frac{4}{7}$.

CHECK Your Progress

Identify the additive inverse and multiplicative inverse for each number.

3A. 1.25 **3B.** $2\frac{1}{2}$

Concepts in Motion
Animation algebra2.com

You can model the Distributive Property using algebra tiles.

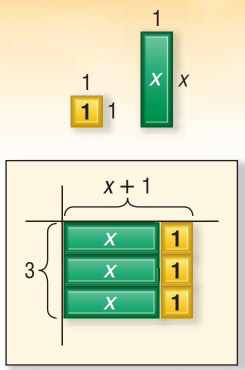
ALGEBRA LAB

Distributive Property

Step 1 A 1-tile is a square that is 1 unit wide and 1 unit long. Its area is 1 square unit. An x-tile is a rectangle that is 1 unit wide and x units long. Its area is x square units.

Step 2 To find the product $3(x + 1)$, model a rectangle with a width of 3 and a length of $x + 1$. Use your algebra tiles to mark off the dimensions on a product mat. Then make the rectangle with algebra tiles.

Step 3 The rectangle has 3 x-tiles and 3 1-tiles. The area of the rectangle is $x + x + x + 1 + 1 + 1$ or $3x + 3$. Thus, $3(x + 1) = 3x + 3$.



MODEL AND ANALYZE

Tell whether each statement is *true* or *false*. Justify your answer with algebra tiles and a drawing.

1. $4(x + 2) = 4x + 2$

2. $3(2x + 4) = 6x + 7$

3. $2(3x + 5) = 6x + 10$

4. $(4x + 1)5 = 4x + 5$

Real-World EXAMPLE

4 **FOOD SERVICE** **A restaurant adds a 20% tip to the bills of parties of 6 or more people. Suppose a server waits on five such tables. The bill without the tip for each party is listed in the table. How much did the server make in tips during this shift?**

Party 1	Party 2	Party 3	Party 4	Party 5
\$185.45	\$205.20	\$195.05	\$245.80	\$262.00

There are two ways to find the total amount of tips received.

Method 1 Multiply each dollar amount by 20% or 0.2 and then add.

$$T = 0.2(185.45) + 0.2(205.20) + 0.2(195.05) + 0.2(245.80) + 0.2(262)$$
$$= 37.09 + 41.04 + 39.01 + 49.16 + 52.40$$
$$= 218.70$$

Method 2 Add all of the bills and then multiply the total by 0.2.

$$T = 0.2(185.45 + 205.20 + 195.05 + 245.80 + 262)$$
$$= 0.2(1093.50)$$
$$= 218.70$$

The server made \$218.70 during this shift.

Notice that both methods result in the same answer.

Real-World Link

Leaving a "tip" began in 18th century English coffee houses and is believed to have originally stood for "To Insure Promptness." Today, the American Automobile Association suggests leaving a 15% tip.

Source: Market Facts, Inc.

CHECK Your Progress

4. Kayla makes \$8 per hour working at a grocery store. The number of hours Kayla worked each day in one week are 3, 2.5, 2, 1, and 4. How much money did Kayla earn this week?

Online **Personal Tutor at** algebra2.com

The properties of real numbers can be used to simplify algebraic expressions.

EXAMPLE Simplify an Expression

5 **Simplify $2(5m + n) + 3(2m - 4n)$.**

$2(5m + n) + 3(2m - 4n)$

$= 2(5m) + 2(n) + 3(2m) - 3(4n)$	Distributive Property
$= 10m + 2n + 6m - 12n$	Multiply.
$= 10m + 6m + 2n - 12n$	Commutative Property (+)
$= (10 + 6)m + (2 - 12)n$	Distributive Property
$= 16m - 10n$	Simplify.

CHECK Your Progress

5. Simplify $3(4x - 2y) - 2(3x + y)$.

CHECK Your Understanding

Example 1 (p. 12)

Name the sets of numbers to which each number belongs.

1. -4 **2.** 45 **3.** $6.\overline{23}$

Example 2 (p. 13)

Name the property illustrated by each question.

4. $\frac{2}{3} \cdot \frac{3}{2} = 1$ **5.** $(a + 4) + 2 = a + (4 + 2)$ **6.** $4x + 0 = 4x$

Example 3 (p. 13)

Identify the additive inverse and multiplicative inverse for each number.

7. -8 **8.** $\frac{1}{3}$ **9.** 1.5

Example 4 (p. 14)

FUND-RAISING For Exercises 10 and 11, use the table.

Catalina is selling candy for $1.50 each to raise money for the band.

10. Write an expression to represent the total amount of money Catalina raised during this week.

11. Evaluate the expression from Exercise 10 by using the Distributive Property.

Catalina's Sales for One Week

Day	Bars Sold
Monday	10
Tuesday	15
Wednesday	12
Thursday	8
Friday	19
Saturday	22
Sunday	31

Example 5 (p. 14)

Simplify each expression.

12. $3(5c + 4d) + 6(d - 2c)$

13. $\frac{1}{2}(16 - 4a) - \frac{3}{4}(12 + 20a)$

Exercises

HOMEWORK HELP

For Exercises	See Examples
14–21	1
22–27	2
28–33	3
34, 35	4
36–43	5

Name the sets of numbers to which each number belongs.

14. $-\frac{2}{9}$ **15.** -4.55 **16.** $-\sqrt{10}$ **17.** $\sqrt{19}$

18. -31 **19.** $\frac{12}{2}$ **20.** $\sqrt{121}$ **21.** $-\sqrt{36}$

Name the property illustrated by each equation.

22. $5a + (-5a) = 0$ **23.** $-6xy + 0 = -6xy$

24. $[5 + (-2)] + (-4) = 5 + [-2 + (-4)]$ **25.** $(2 + 14) + 3 = 3 + (2 + 14)$

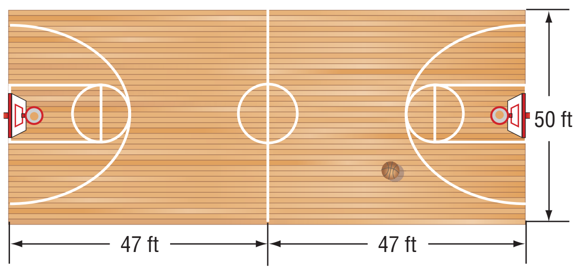
26. $\left(1\frac{2}{7}\right)\left(\frac{7}{9}\right) = 1$ **27.** $2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3}$

Identify the additive inverse and multiplicative inverse for each number.

28. -10 **29.** 2.5 **30.** -0.125

31. $-\frac{5}{8}$ **32.** $\frac{4}{3}$ **33.** $-4\frac{3}{5}$

34. BASKETBALL Illustrate the Distributive Property by writing two expressions for the area of the NCAA basketball court. Then find the area of the basketball court.



35. BAKING Mitena is making two types of cookies. The first recipe calls for $2\frac{1}{4}$ cups of flour, and the second calls for $1\frac{1}{8}$ cups of flour. If she wants to make 3 batches of the first recipe and 2 batches of the second recipe, how many cups of flour will she need? Use the properties of real numbers to show how Mitena could compute this amount mentally. Justify each step.

Simplify each expression.

36. $7a + 3b - 4a - 5b$

37. $3x + 5y + 7x - 3y$

38. $3(15x - 9y) + 5(4y - x)$

39. $2(10m - 7a) + 3(8a - 3m)$

40. $8(r + 7t) - 4(13t + 5r)$

41. $4(14c - 10d) - 6(d + 4c)$

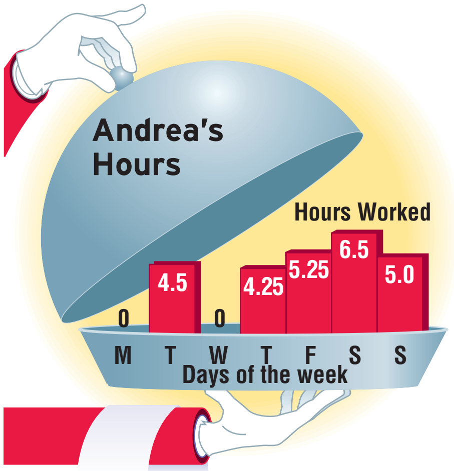
42. $4(0.2m - 0.3n) - 6(0.7m - 0.5n)$

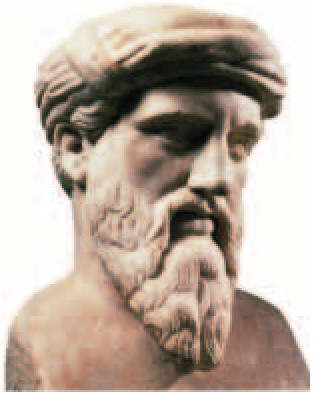
43. $7(0.2p + 0.3q) + 5(0.6p - q)$

WORK **For Exercises 44 and 45, use the information below and in the graph.**
Andrea works in a restaurant and is paid every two weeks.

44. If Andrea earns \$6.50 an hour, illustrate the Distributive Property by writing two expressions representing Andrea's pay last week.

45. Find the mean or average number of hours Andrea worked each day, to the nearest tenth of an hour. Then use this average to predict her pay for a two-week pay period.





Real-World Link
Pythagoras (572–497 B.C.) was a Greek philosopher whose followers came to be known as the Pythagoreans. It was their knowledge of what is called the Pythagorean Theorem that led to the first discovery of irrational numbers.

Source: *A History of Mathematics*

NUMBER THEORY **For Exercises 46–49, use the properties of real numbers to answer each question.**

46. If $m + n = m$, what is the value of n?

47. If $m + n = 0$, what is the value of n? What is n's relationship to m?

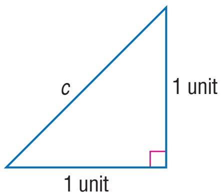
48. If $mn = 1$, what is the value of n? What is n's relationship to m?

49. If $mn = m$ and $m \neq 0$, what is the value of n?

MATH HISTORY **For Exercises 50–52, use the following information.**
The Greek mathematician Pythagoras believed that all things could be described by numbers. By *number* he meant a positive integer.

50. To what set of numbers was Pythagoras referring when he spoke of *numbers*?

51. Use the formula $c = \sqrt{2s^2}$ to calculate the length of the hypotenuse c, or longest side, of this right triangle using s, the length of one leg.



52. Explain why Pythagoras could not find a "number" for the value of c.

Name the sets of numbers to which each number belongs.

53. 0

54. $\frac{3\pi}{2}$

55. $-2\sqrt{7}$

56. Name the sets of numbers to which all of the following numbers belong. Then arrange the numbers in order from least to greatest.

$$2.\overline{49},\ 2.4\overline{9},\ 2.4,\ 2.49,\ 2.\overline{9}$$

EXTRA PRACTICE
See pages 891, 926.
Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

OPEN ENDED **Give an example of a number that satisfies each condition.**

57. integer, but not a natural number

58. integer with a multiplicative inverse that is an integer

CHALLENGE Determine whether each statement is *true* or *false*. If *false*, give a counterexample. A **counterexample** is a specific case that shows that a statement is false.

59. Every whole number is an integer.

60. Every integer is a whole number.

61. Every real number is irrational.

62. Every integer is a rational number.

63. **REASONING** Is the Distributive Property also true for division? In other words, does $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$, $a \neq 0$? If so, give an example and explain why it is true. If not true, give a counterexample.

64. ***Writing in Math*** Use the information about coupons on page 11 to explain how the Distributive Property is useful in calculating store savings. Include an explanation of how the Distributive Property could be used to calculate the coupon savings listed on a grocery receipt.

STANDARDIZED TEST PRACTICE

65. **ACT/SAT** If a and b are natural numbers, then which of the following must also be a natural number?

I. $a - b$ **II.** ab **III.** $\frac{a}{b}$

A I only

B II only

C III only

D I and II only

66. **REVIEW** Which equation is equivalent to $4(9 - 3x) = 7 - 2(6 - 5x)$?

F $8x = 41$

G $8x = 24$

H $22x = 41$

J $22x = 24$

Spiral Review

Evaluate each expression. (Lesson 1-1)

67. $9(4 - 3)^5$

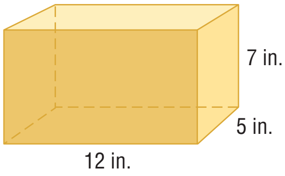
68. $5 + 9 \div 3(3) - 8$

Evaluate each expression if $a = -5$, $b = 0.25$, $c = \frac{1}{2}$, and $d = 4$. (Lesson 1-1)

69. $a + 2b - c$

70. $b + 3(a + d)^3$

71. **GEOMETRY** The formula for the surface area SA of a rectangular prism is $SA = 2\ell w + 2\ell h + 2wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the surface area of the rectangular prism. (Lesson 1-1)



GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $a = 2$, $b = -\frac{3}{4}$, and $c = 1.8$.** (Lesson 1-1)

72. $8b - 5$

73. $\frac{2}{5}b + 1$

74. $1.5c - 7$

75. $-9(a - 6)$